

# High School Content Expectations



## MATHEMATICS

- Quantitative Literacy and Logic
- Algebra and Functions
- Geometry and Trigonometry
- Statistics and Probability

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# Welcome to Michigan's High School Mathematics Content Standards and Expectations

## Why Develop Content Standards and Expectations for High School?

In 2004, the Michigan Department of Education embraced the challenge to initiate a “high school redesign” project. Since then, the national call to create more rigorous learning for high school students has become a major priority for state leaders across the country. The Cherry Commission Report highlighted several goals for Michigan including the development of high school content expectations that reflect both a rigorous and a relevant curricular focus. Dovetailing with this call to “curricular action” is Michigan’s legislative change in high school assessment. The Michigan Merit Exam, based on rigorous high school learning standards, is to be fully implemented by 2007.

Given these two catalysts, the Michigan Department of Education’s Office of School Improvement led the development of high school content expectations for English Language Arts and Mathematics. Content area work groups of academicians chaired by a nationally known scholar in the respective field, were commissioned to conduct a scholarly review and identify content standards and expectations. These content standards and expectations have gone through an extensive field and national review and are presented to educators in this document.

## An Overview

The expectations contained in this document reflect best practices and current research in the teaching and learning of mathematics. They build from the *Michigan Mathematics Curriculum Framework Standards and Benchmarks* (1996), the *Career and Employability Skills Content Standards and Benchmarks* (2001), and extend the *Michigan K-8 Mathematics Grade Level Content Expectations* (2004) as appropriate for grades 9-12. These standards and expectations represent a vision for a rigorous and relevant high school experience for all Michigan students over the next five to ten years. Special attention has been paid to national research and support for the skills that prepare students for successful post-secondary engagement and the workplace.

The standards and expectations are closely aligned with national standards as described in ACT’s *College Readiness Standards*®, American Diploma Project’s *Ready or Not: Creating a High School Diploma That Counts* (2004), the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* (2000), and the National Assessment Governing Board’s *Mathematics Framework* for the 2003 *National Assessment of Educational Progress* (NAEP). Students whose work is guided by these standards and expectations will be prepared both for college and for the workplace.

## Curriculum and Assessment

This document is intended to support conversations at the school and district level that result in rigorous and relevant curriculum that incorporates these content expectations.

**As stakeholders (i.e., teachers, administrators, school board members, parents, community members, students, local legislative representatives) work with these standards, they should consider the following questions:**

- How are these content standards and expectations reflected in our curriculum and instruction already?
- Where do we need to strengthen our curriculum and instruction to more fully realize the intent of these standards and expectations?
- What opportunities do these standards and expectations present to develop new and strengthen existing curriculum, leading to instructional excellence?
- How do we implement these standards and expectations taking into account what we know about our students, school, and community?
- How will we assess the effectiveness with which our students and schools are meeting these standards and content expectations?
- How can we use school-based assessments (e.g., student portfolios, school-based writing assessments, teacher or classroom research, district-level assessments) to make data-driven decisions about teaching and learning?

Through conversations about questions such as these, and building upon the multitude of existing strengths in our current high schools, voices of all stakeholders will participate in the important and continuing process of shaping instructional excellence in Michigan schools and preparing students in Michigan schools for college and the workplace.

## Mathematics

Mathematical understandings and skills are essential elements for meaningful participation in the global information society. US expectations in mathematics for high school students have not kept pace with expectations in high-achieving countries around the world. And, expectations about who can do mathematics in the US have led to inequitable and unacceptably low opportunities to learn for students living in poor and urban communities. In Michigan, the K-8 Mathematics Grade Level Expectations represent a major step forward in raising expectations in mathematics for all students. These high school expectations assume the ambitious foundation of the K-8 GLCEs and are intended to equip all students with a solid background for continued postsecondary study in any area, as well as with skills and knowledge essential for the workplace. It is essential to hold high expectations in mathematics for all students for completion of high school, whether they will enter the workforce or go on to postsecondary education.

The high school mathematics content expectations are organized in four strands: Quantitative Literacy and Logic, Algebra and Functions, Geometry and Trigonometry, and Statistics and Probability. The topics within each strand have been arranged to show mathematical growth and to illustrate mathematical trajectories of ideas that build on one another, when possible. The expectations in these four strands, are not mapped into course arrangements in this document. Such mapping, whether to traditional course titles like Algebra I, Geometry, or Algebra II, or into courses that integrate the material, is a complex process.

### Decisions about the inclusion of topics were based on the following five criteria:

- how well the topic connects to other mathematical areas
- the mathematical centrality of the topic
- the standing of the topic as a cultural accomplishment
- the relevance of the topic for secondary school students
- the importance of the topic in the workplace or for informed citizenship

There is a strong emphasis on mathematical reasoning throughout all of these strands. It is also important for high school students to become successful in applying mathematical concepts and processes to solve complex problems. Technological advances affect what is possible to learn, and what is necessary to learn, in high school mathematics, and these expectations reflect this trend.

These four strands are fundamentally interconnected and also arranged to reflect the sequencing and emphases in the mathematical ideas that are central to high school.

## Understanding the Organizational Structure

The expectations in this document are divided into four strands with multiple standards within each, as shown below. The skills and content addressed in these standards will, in practice, be woven together into a coherent, integrated Mathematics curriculum. The standards are comprehensive and are meant to be used as a guide to curriculum development.

<b>STRAND 1</b> <b>Quantitative Literacy and Logic (L)</b>	<b>STRAND 2</b> <b>Algebra and Functions (A)</b>	<b>STRAND 3</b> <b>Geometry and Trigonometry (G)</b>	<b>STRAND 4</b> <b>Statistics and Probability (S)</b>
<b>STANDARDS (and number of core expectations in each standard)</b>			
L1: Reasoning About Numbers, Systems and Quantitative Situations (13) L2: Calculation, Algorithms, and Estimation (13) L3: Mathematical Reasoning, Logic, and Proof (10)	A1: Expressions, Equations, and Inequalities (16) A2: Functions (16) A3: Families of Functions (27)	G1: Figures and Their Properties (29) G2: Relationships Between Figures (10) G3: Transformations of Figures in the Plane (5)	S1: Univariate Data—Examining Distributions (9) S2: Bivariate Data—Examining Relationships (6) S3: Samples, Surveys, and Experiments (3) S4: Probability Models and Probability Calculation (4)
<i>Recommended Quantitative Literacy and Logic Expectations (3)</i>	<i>Recommended Algebra and Functions Expectations (5)</i>	<i>Recommended Geometry and Trigonometry Expectations (3)</i>	<i>Recommended Statistics and Probability Expectations (6)</i>

## Core and Recommended Expectations

The expectations in this document represent what all Michigan high school graduates should know and be able to do in mathematics. With a focused and coherent set of **required core expectations**, teachers can provide both the breadth of mathematical experiences required for students to succeed in an increasingly competitive world economy, and also provide the depth required for mastery of fundamental mathematical ideas. There should be far less of the review and revisiting of topics that is typical in the high school mathematics curriculum. With a deep understanding of these expectations, students will make connections among fundamental mathematical ideas, and will be well-situated to use their mathematical knowledge and quantitative skills across the curriculum.

At the end of each strand, a set of **recommended expectations** is listed. These extensions represent content that is desirable and valuable for all students, but attention to these items should not displace or dilute the curricular emphasis of any of the core expectations. Teachers are encouraged to incorporate the recommended expectations into their instruction when their students have a solid foundation and are ready for enrichment or advanced learning.

## Updates

This 11/07 version of the High School Mathematics Content Expectations updates the original 8/06 version by:

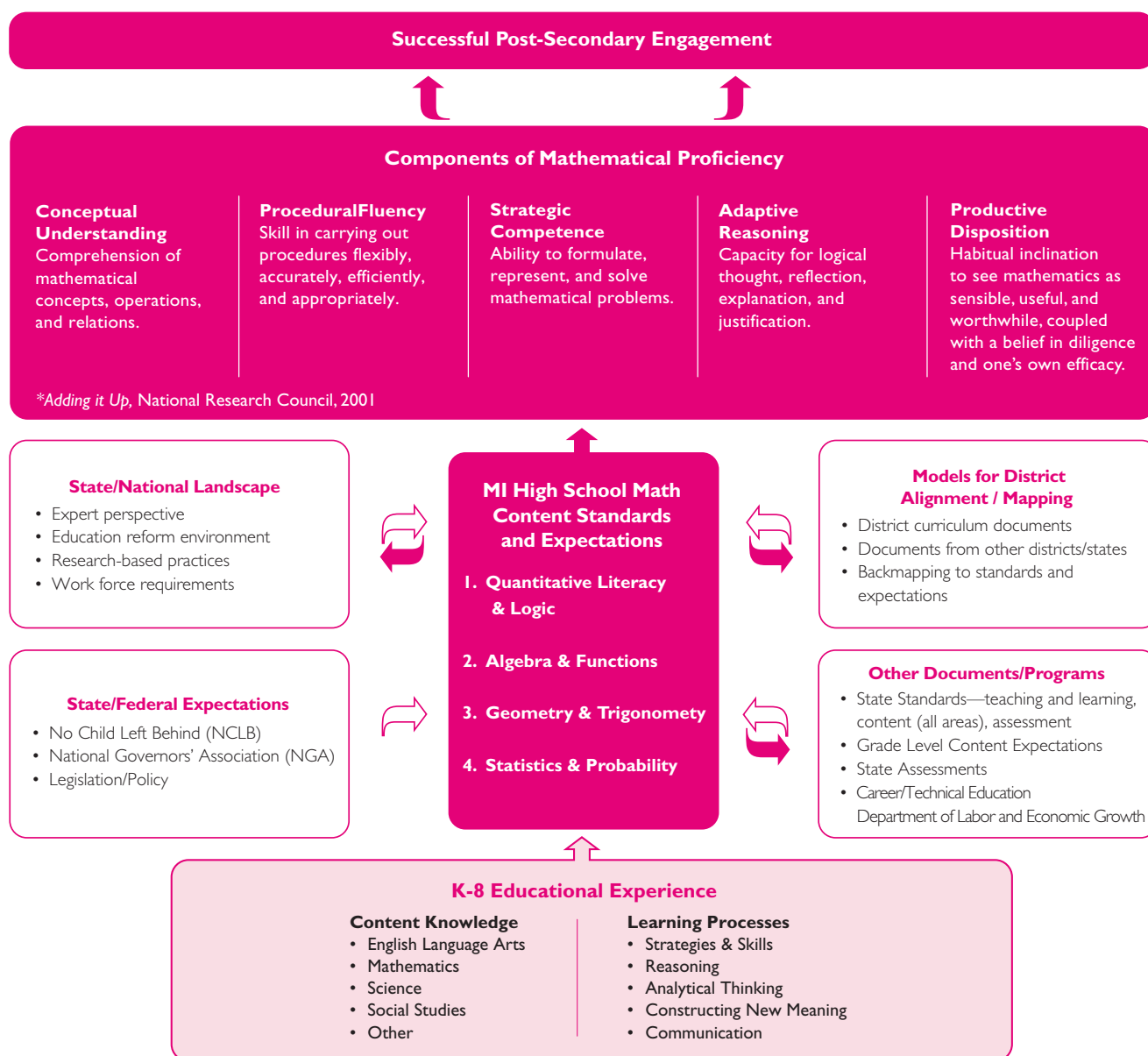
- Moving Standard A3-Mathematical Modeling to A2.4. All the topics dealing with individual function families are now in A3.
- Combining Standards L2 and L3 into one standard; Standard L4 is now L3.
- Making minor changes to some expectations to improve understanding.
- Removing examples from the expectations. More extensive examples can be found in the clarification companion documents.

## Preparing Students for Successful Post-Secondary Engagement

As educators use these standards and expectations to develop rigorous and relevant units of instruction, powerful and engaging learning activities, and challenging high school curricula, it is critical to keep in mind that content knowledge alone will not provide adequate preparation for success in entry-level university courses or entry-level positions in today's workforce.

Successful post-secondary engagement requires that students must be able to apply knowledge in new situations; to solve problems by generating new ideas; to make connections between what they read and hear in class, the world around them, and the future; and through their work, develop leadership qualities while still in high school.

Therefore, educators must model for and develop in students the cognitive skills and habits of mind that will result in mathematical proficiency and successful post-secondary engagement.





## STRAND 1: QUANTITATIVE LITERACY AND LOGIC (L)

*"In an increasingly complex world, adults are challenged to apply sophisticated quantitative knowledge and reasoning in their professional and personal lives. The technological demands of the workplace, the abundance of data in the political and public policy context, and the array of information involved in making personal and family decisions of all types necessitate an unprecedented facility not only with fundamental mathematical, statistical, and computing ideas and processes, but with higher-order abilities to apply and integrate those ideas and processes in a range of areas."*<sup>1</sup>

The Michigan Grade Level Content Expectations in Mathematics for grades K-8 prescribe a thorough treatment of number, including strong emphasis on computational fluency and understanding of number concepts, to be completed largely by the sixth grade. The expectations in this Quantitative Literacy and Logic strand provide a definition of secondary school quantitative literacy for all students and emphasize the importance of logic as part of mathematics and in everyday life. They assume fluency (that is, efficiency and accuracy) in calculation with the basic number operations involving rational numbers in all forms (including percentages and decimals), without calculators. Mathematical reasoning and logic are at the heart of the study of mathematics. As students progress through elementary and middle school, they increasingly are asked to explain and justify the thinking underlying their work. In high school, students peel away the contexts and study the language and thought patterns of formal mathematical reasoning. By learning logic and by constructing arguments and proofs, students will strengthen not only their knowledge and facility with mathematics, but also their ways of thinking in other areas of study and in their daily lives.

Connections and applications of number ideas and logic to other areas of mathematics, such as algebra, geometry, and statistics, are emphasized in this strand. Number representations and properties extend from the rational numbers into the real and complex numbers, as well as to other systems that students will encounter both in the workplace and in more advanced mathematics. The expectations for calculation, algorithms and estimation reflect important uses of number in a range of real-life situations. Ideas about measurement and precision tie closely to geometry.

<sup>1</sup> Estrly, D. & Ferrini-Mundy, J. (January, 2005). Quantitative Literacy Task Force Final Report and Recommendations. East Lansing, Michigan State University

### STANDARD L1: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE SITUATIONS

Based on their knowledge of the properties of arithmetic, students understand and reason about numbers, number systems, and the relationships between them. They represent quantitative relationships using mathematical symbols, and interpret relationships from those representations.

#### L1.1 Number Systems and Number Sense

- L1.1.1** Know the different properties that hold in different number systems and recognize that the applicable properties change in the transition from the positive integers to all integers, to the rational numbers, and to the real numbers.
- L1.1.2** Explain why the multiplicative inverse of a number has the same sign as the number, while the additive inverse of a number has the opposite sign.
- L1.1.3** Explain how the properties of associativity, commutativity, and distributivity, as well as identity and inverse elements, are used in arithmetic and algebraic calculations.
- L1.1.4** Describe the reasons for the different effects of multiplication by, or exponentiation of, a positive number by a number less than 0, a number between 0 and 1, and a number greater than 1.
- L1.1.5** Justify numerical relationships
- L1.1.6** Explain the importance of the irrational numbers  $\sqrt{2}$  and  $\sqrt{3}$  in basic right triangle trigonometry, and the importance of  $\pi$  because of its role in circle relationships.

## STRAND 1: QUANTITATIVE LITERACY AND LOGIC (CONT.)

### L1.2 Representations and Relationships

- L1.2.1** Use mathematical symbols to represent quantitative relationships and situations.
- L1.2.2** Interpret representations that reflect absolute value relationships in such contexts as error tolerance.
- L1.2.3** Use vectors to represent quantities that have magnitude and direction, interpret direction and magnitude of a vector numerically, and calculate the sum and difference of two vectors.
- L1.2.4** Organize and summarize a data set in a table, plot, chart, or spreadsheet; find patterns in a display of data; understand and critique data displays in the media.

### L1.3 Counting and Probabilistic Reasoning

- L1.3.1** Describe, explain, and apply various counting techniques; relate combinations to Pascal's triangle; know when to use each technique.
- L1.3.2** Define and interpret commonly used expressions of probability.
- L1.3.3** Recognize and explain common probability misconceptions such as "hot streaks" and "being due."

## STANDARD L2: CALCULATION, ALGORITHMS, AND ESTIMATION

Students calculate fluently, estimate proficiently, and describe and use algorithms in appropriate situations (e.g., approximating solutions to equations). They understand the basic ideas of iteration and algorithms.

### L2.1 Calculation Using Real and Complex Numbers

- L2.1.1** Explain the meaning and uses of weighted averages.
- L2.1.2** Calculate fluently with numerical expressions involving exponents; use the rules of exponents; evaluate numerical expressions involving rational and negative exponents; transition easily between roots and exponents.
- L2.1.3** Explain the exponential relationship between a number and its base 10 logarithm and use it to relate rules of logarithms to those of exponents in expressions involving numbers.
- L2.1.4** Know that the complex number  $i$  is one of two solutions to  $x^2 = -1$ .
- L2.1.5** Add, subtract, and multiply complex numbers; use conjugates to simplify quotients of complex numbers.

### L2.2 Sequences and Iteration

- L2.2.1** Find the  $n$ th term in arithmetic, geometric, or other simple sequences.
- L2.2.2** Compute sums of finite arithmetic and geometric sequences.
- L2.2.3** Use iterative processes in such examples as computing compound interest or applying approximation procedures.

### L2.3 Measurement Units, Calculations, and Scales

- L2.3.1** Convert units of measurement within and between systems; explain how arithmetic operations on measurements affect units, and carry units through calculations correctly.
- L2.3.2** Describe and interpret logarithmic relationships in such contexts as the Richter scale, the pH scale, or decibel measurements; solve applied problems.



## STRAND 1: QUANTITATIVE LITERACY AND LOGIC (CONT.)

### L2.4 Understanding Error

- L2.4.1** Determine what degree of accuracy is reasonable for measurements in a given situation; express accuracy through use of significant digits, error tolerance, or percent of error; describe how errors in measurements are magnified by computation; recognize accumulated error in applied situations.
- L2.4.2** Describe and explain round-off error, rounding, and truncating.
- L2.4.3** Know the meaning of and interpret statistical significance, margin of error, and confidence level.

## STANDARD L3: MATHEMATICAL REASONING, LOGIC, AND PROOF

Students understand mathematical reasoning as being grounded in logic and proof and can distinguish mathematical arguments from other types of arguments. They can interpret arguments made about quantitative situations in the popular media. Students know the language and laws of logic and can apply them in both mathematical and everyday settings. They write proofs using direct and indirect methods and use counterexamples appropriately to show that statements are false.

### L3.1 Mathematical Reasoning

- L3.1.1** Distinguish between inductive and deductive reasoning, identifying and providing examples of each.
- L3.1.2** Differentiate between statistical arguments (statements verified empirically using examples or data) and logical arguments based on the rules of logic.
- L3.1.3** Define and explain the roles of axioms (postulates), definitions, theorems, counterexamples, and proofs in the logical structure of mathematics. Identify and give examples of each.

### L3.2 Language and Laws of Logic

- L3.2.1** Know and use the terms of basic logic.
- L3.2.2** Use the connectives “not,” “and,” “or,” and “if..., then,” in mathematical and everyday settings. Know the truth table of each connective and how to logically negate statements involving these connectives.
- L3.2.3** Use the quantifiers “there exists” and “all” in mathematical and everyday settings and know how to logically negate statements involving them.
- L3.2.4** Write the converse, inverse, and contrapositive of an “if..., then...” statement. Use the fact, in mathematical and everyday settings, that the contrapositive is logically equivalent to the original, while the inverse and converse are not.

### L3.3 Proof

- L3.3.1** Know the basic structure for the proof of an “if..., then...” statement (assuming the hypothesis and ending with the conclusion) and that proving the contrapositive is equivalent.
- L3.3.2** Construct proofs by contradiction. Use counterexamples, when appropriate, to disprove a statement.
- L3.3.3** Explain the difference between a necessary and a sufficient condition within the statement of a theorem. Determine the correct conclusions based on interpreting a theorem in which necessary or sufficient conditions in the theorem or hypothesis are satisfied.

**RECOMMENDED:**

- \***L1.2.5** Read and interpret representations from various technological sources, such as contour or isobar diagrams.
- \***L2.1.7** Understand the mathematical bases for the differences among voting procedures.
- \***L2.2.4** Compute sums of infinite geometric sequences.

## STRAND 2: ALGEBRA AND FUNCTIONS

(A)

In the middle grades, students see the progressive generalization of arithmetic to algebra. They learn symbolic manipulation skills and use them to solve equations. They study simple forms of elementary polynomial functions such as linear, quadratic, and power functions as represented by tables, graphs, symbols, and verbal descriptions.

In high school, students continue to develop their “symbol sense” by examining expressions, equations, and functions, and applying algebraic properties to solve equations. They construct a conceptual framework for analyzing any function and, using this framework, they revisit the functions they have studied before in greater depth. By the end of high school, their catalog of functions will encompass linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric functions. They will be able to reason about functions and their properties and solve multistep problems that involve both functions and equation-solving. Students will use deductive reasoning to justify algebraic processes as they solve equations and inequalities, as well as when transforming expressions.

This rich learning experience in Algebra will provide opportunities for students to understand both its structure and its applicability to solving real-world problems. Students will view algebra as a tool for analyzing and describing mathematical relationships, and for modeling problems that come from the workplace, the sciences, technology, engineering, and mathematics.

### STANDARD A1: EXPRESSIONS, EQUATIONS, AND INEQUALITIES

Students recognize, construct, interpret, and evaluate expressions. They fluently transform symbolic expressions into equivalent forms. They determine appropriate techniques for solving each type of equation, inequality, or system of equations, apply the techniques correctly to solve, justify the steps in the solutions, and draw conclusions from the solutions. They know and apply common formulas.

#### A1.1 Construction, Interpretation, and Manipulation of Expressions

- A1.1.1** Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.
- A1.1.2** Know the definitions and properties of exponents and roots transition fluently between them, and apply them in algebraic expressions.
- A1.1.3** Factor algebraic expressions using, for example, greatest common factor, grouping, and the special product identities.
- A1.1.4** Add, subtract, multiply, and simplify polynomials and rational expressions.
- A1.1.5** Divide a polynomial by a monomial.
- A1.1.6** Transform exponential and logarithmic expressions into equivalent forms using the properties of exponents and logarithms, including the inverse relationship between exponents and logarithms.

#### A1.2 Solutions of Equations and Inequalities

- A1.2.1** Write equations and inequalities with one or two variables to represent mathematical or applied situations, and solve.
- A1.2.2** Associate a given equation with a function whose zeros are the solutions of the equation.
- A1.2.3** Solve linear and quadratic equations and inequalities including systems of up to three linear equations with three unknowns. Justify steps in the solution, and apply the quadratic formula appropriately.
- A1.2.4** Solve absolute value equations and inequalities, and justify steps in the solution.
- A1.2.5** Solve polynomial equations and equations involving rational expressions, and justify steps in the solution.

## STRAND 2: ALGEBRA AND FUNCTIONS (CONT.)

- A1.2.6** Solve power equations and equations including radical expressions, justify steps in the solution, and explain how extraneous solutions may arise.
- A1.2.7** Solve exponential and logarithmic equations, and justify steps in the solution.
- A1.2.8** Solve an equation involving several variables (with numerical or letter coefficients) for a designated variable. Justify steps in the solution.
- A1.2.9** Know common formulas and apply appropriately in contextual situations.
- A1.2.10** Use special values of the inverse trigonometric functions to solve trigonometric equations over specific intervals.

### STANDARD A2: FUNCTIONS

Students understand functions, their representations, and their attributes. They perform transformations, combine and compose functions, and find inverses. Students classify functions and know the characteristics of each family. They work with functions with real coefficients fluently. Students construct or select a function to model a real-world situation in order to solve applied problems. They draw on their knowledge of families of functions to do so.

#### A2.1 Definitions, Representations, and Attributes of Functions

- A2.1.1** Determine whether a relationship (given in contextual, symbolic, tabular, or graphical form) is a function and identify its domain and range.
- A2.1.2** Read, interpret, and use function notation and evaluate a function at a value in its domain.
- A2.1.3** Represent functions in symbols, graphs, tables, diagrams, or words and translate among representations.
- A2.1.4** Recognize that functions may be defined by different expressions over different intervals of their domains; such functions are piecewise-defined.
- A2.1.5** Recognize that functions may be defined recursively. Compute values of and graph simple recursively defined functions.
- A2.1.6** Identify the zeros of a function, the intervals where the values of a function are positive or negative, and describe the behavior of a function as  $x$  approaches positive or negative infinity, given the symbolic and graphical representations.
- A2.1.7** Identify and interpret the key features of a function from its graph or its formula(e).

#### A2.2 Operations and Transformations

- A2.2.1** Combine functions by addition, subtraction, multiplication, and division.
- A2.2.2** Apply given transformations to basic functions and represent symbolically.
- A2.2.3** Recognize whether a function (given in tabular or graphical form) has an inverse and recognize simple inverse pairs.

#### A2.3 Representations of Functions

- A2.3.1** Identify a function as a member of a family of functions based on its symbolic or graphical representation; recognize that different families of functions have different asymptotic behavior.

## STRAND 2: ALGEBRA AND FUNCTIONS (CONT.)

**A2.3.2** Describe the tabular pattern associated with functions having constant rate of change (linear); or variable rates of change.

**A2.3.3** Write the general symbolic forms that characterize each family of functions.

### A2.4 Models of Real-world Situations Using Families of Functions

**A2.4.1** Identify the family of function best suited for modeling a given real-world situation.

**A2.4.2** Adapt the general symbolic form of a function to one that fits the specification of a given situation by using the information to replace arbitrary constants with numbers.

**A2.4.3** Using the adapted general symbolic form, draw reasonable conclusions about the situation being modeled.

## STANDARD A3: FAMILIES OF FUNCTIONS

Students study the symbolic and graphical forms of each function family. By recognizing the unique characteristics of each family, they can use them as tools for solving problems or for modeling real-world situations.

### A3.1 Lines and Linear Functions

**A3.1.1** Write the symbolic forms of linear functions (standard, point-slope, and slope-intercept) given appropriate information, and convert between forms.

**A3.1.2** Graph lines (including those of the form  $x = h$  and  $y = k$ ) given appropriate information.

**A3.1.3** Relate the coefficients in a linear function to the slope and  $x$ - and  $y$ -intercepts of its graph.

**A3.1.4** Find an equation of the line parallel or perpendicular to given line, through a given point; understand and use the facts that non-vertical parallel lines have equal slopes, and that non-vertical perpendicular lines have slopes that multiply to give  $-1$ .

### A3.2 Exponential and Logarithmic Functions

**A3.2.1** Write the symbolic form and sketch the graph of an exponential function given appropriate information.

**A3.2.2** Interpret the symbolic forms and recognize the graphs of exponential and logarithmic functions; recognize the logarithmic function as the inverse of the exponential function.

**A3.2.3** Apply properties of exponential and logarithmic functions.

**A3.2.4** Understand and use the fact that the base of an exponential function determines whether the function increases or decreases and understand how the base affects the rate of growth or decay.

**A3.2.5** Relate exponential and logarithmic functions to real phenomena, including half-life and doubling time.

### A3.3 Quadratic Functions

**A3.3.1** Write the symbolic form and sketch the graph of a quadratic function given appropriate information.

**A3.3.2** Identify the elements of a parabola (vertex, axis of symmetry, direction of opening) given its symbolic form or its graph, and relate these elements to the coefficient(s) of the symbolic form of the function.

**A3.3.3** Convert quadratic functions from standard to vertex form by completing the square.

**A3.3.4** Relate the number of real solutions of a quadratic equation to the graph of the associated quadratic function.

**A3.3.5** Express quadratic functions in vertex form to identify their maxima or minima, and in factored form to identify their zeros.

## STRAND 2: ALGEBRA AND FUNCTIONS (CONT.)

### A3.4 Power Functions

- A3.4.1** Write the symbolic form and sketch the graph of power functions.
- A3.4.2** Express direct and inverse relationships as functions and recognize their characteristics.
- A3.4.3** Analyze the graphs of power functions, noting reflectional or rotational symmetry.

### A3.5 Polynomial Functions

- A3.5.1** Write the symbolic form and sketch the graph of simple polynomial functions.
- A3.5.2** Understand the effects of degree, leading coefficient, and number of real zeros on the graphs of polynomial functions of degree greater than 2.
- A3.5.3** Determine the maximum possible number of zeros of a polynomial function, and understand the relationship between the x-intercepts of the graph and the factored form of the function.

### A3.6 Rational Functions

- A3.6.1** Write the symbolic form and sketch the graph of simple rational functions.
- A3.6.2** Analyze graphs of simple rational functions and understand the relationship between the zeros of the numerator and denominator and the function's intercepts, asymptotes, and domain.

### A3.7 Trigonometric Functions

- A3.7.1** Use the unit circle to define sine and cosine; approximate values of sine and cosine; use sine and cosine to define the remaining trigonometric functions; explain why the trigonometric functions are periodic.
- A3.7.2** Use the relationship between degree and radian measures to solve problems.
- A3.7.3** Use the unit circle to determine the exact values of sine and cosine, for integer multiples of  $\pi/6$  and  $\pi/4$ .
- A3.7.4** Graph the sine and cosine functions; analyze graphs by noting domain, range, period, amplitude, and location of maxima and minima.
- A3.7.5** Graph transformations of basic trigonometric functions (involving changes in period, amplitude, and midline) and understand the relationship between constants in the formula and the transformed graph.

#### RECOMMENDED:

- \*A1.1.7** Transform trigonometric expressions into equivalent forms using basic identities such as  $\sin^2 \theta + \cos^2 \theta = 1$  and  $\tan^2 \theta + 1 = \sec^2 \theta$
- \*A2.2.4** If a function has an inverse, find the expression(s) for the inverse.
- \*A2.2.5** Write an expression for the composition of one function with another; recognize component functions when a function is a composition of other functions.
- \*A2.2.6** Know and interpret the function notation for inverses and verify that two functions are inverses using composition.
- \*A2.4.4** Use methods of linear programming to represent and solve simple real-life problems.



## STRAND 3: GEOMETRY AND TRIGONOMETRY

(G)

In Grades K–5, students study figures such as triangles, rectangles, circles, rectangular solids, cylinders, and spheres. They examine similarities and differences between geometric shapes. They learn to quantify geometric figures by measuring and calculating lengths, angles, areas and volumes. In Grades 6–8, students broaden their understanding of area and volume and develop the basic concepts of congruence, similarity, symmetry and the Pythagorean Theorem. They apply these ideas to solve geometric problems, including ones related to the real world.

In Grades 9–12, students see geometry developed as a coherent, structured subject. They use the geometric skills and ideas introduced earlier, such as congruence and similarity, to solve a wide variety of problems. There is an emphasis on the importance of clear language and on learning to construct geometric proofs. In this process, students build geometric intuition and facility at deductive reasoning. They use elements of logic and reasoning as described in the Quantitative Literacy and Logic strand, including both direct and indirect proof presented in narrative form. They begin to use new techniques, including transformations and trigonometry. They apply these ideas to solve complex problems about two- and three-dimensional figures, again including ones related to the real world. Their spatial visualization skills will be developed through the study of the relationships between two- and three-dimensional shapes.

### STANDARD G1: FIGURES AND THEIR PROPERTIES

Students represent basic geometric figures, polygons, and conic sections and apply their definitions and properties in solving problems and justifying arguments, including constructions and representations in the coordinate plane. Students represent three-dimensional figures, understand the concepts of volume and surface area, and use them to solve problems. They know and apply properties of common three-dimensional figures.

#### G1.1 Lines and Angles; Basic Euclidean and Coordinate Geometry

- G1.1.1** Solve multistep problems and construct proofs involving vertical angles, linear pairs of angles supplementary angles, complementary angles, and right angles.
- G1.1.2** Solve multistep problems and construct proofs involving corresponding angles, alternate interior angles, alternate exterior angles, and same-side (consecutive) interior angles.
- G1.1.3** Perform and justify constructions, including midpoint of a line segment and bisector of an angle, using straightedge and compass.
- G1.1.4** Given a line and a point, construct a line through the point that is parallel to the original line using straightedge and compass. Given a line and a point, construct a line through the point that is perpendicular to the original line. Justify the steps of the constructions.
- G1.1.5** Given a line segment in terms of its endpoints in the coordinate plane, determine its length and midpoint.
- G1.1.6** Recognize Euclidean geometry as an axiom system. Know the key axioms and understand the meaning of and distinguish between undefined terms, axioms, definitions, and theorems.

#### G1.2 Triangles and Their Properties

- G1.2.1** Prove that the angle sum of a triangle is  $180^\circ$  and that an exterior angle of a triangle is the sum of the two remote interior angles.

## STRAND 3: GEOMETRY AND TRIGONOMETRY (CONT.)

- G1.2.2** Construct and justify arguments and solve multistep problems involving angle measure, side length, perimeter, and area of all types of triangles.
- G1.2.3** Know a proof of the Pythagorean Theorem, and use the Pythagorean Theorem and its converse to solve multistep problems.
- G1.2.4** Prove and use the relationships among the side lengths and the angles of  $30^\circ$ -  $60^\circ$ -  $90^\circ$  triangles and  $45^\circ$ -  $45^\circ$ -  $90^\circ$  triangles.
- G1.2.5** Solve multistep problems and construct proofs about the properties of medians, altitudes, perpendicular bisectors to the sides of a triangle, and the angle bisectors of a triangle. Using a straightedge and compass, construct these lines.

### G1.3 Triangles and Trigonometry

- G1.3.1** Define the sine, cosine, and tangent of acute angles in a right triangle as ratios of sides. Solve problems about angles, side lengths, or areas using trigonometric ratios in right triangles.
- G1.3.2** Know and use the Law of Sines and the Law of Cosines and use them to solve problems. Find the area of a triangle with sides  $a$  and  $b$  and included angle  $\theta$  using the formula  $\text{Area} = (1/2) ab \sin \theta$ .
- G1.3.3** Determine the exact values of sine, cosine, and tangent for  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ , and their integer multiples and apply in various contexts.

### G1.4 Quadrilaterals and Their Properties

- G1.4.1** Solve multistep problems and construct proofs involving angle measure, side length, diagonal length, perimeter, and area of squares, rectangles, parallelograms, kites, and trapezoids.
- G1.4.2** Solve multistep problems and construct proofs involving quadrilaterals using Euclidean methods or coordinate geometry.
- G1.4.3** Describe and justify hierarchical relationships among quadrilaterals.
- G1.4.4** Prove theorems about the interior and exterior angle sums of a quadrilateral.

### G1.5 Other Polygons and Their Properties

- G1.5.1** Know and use subdivision or circumscription methods to find areas of polygons.
- G1.5.2** Know, justify, and use formulas for the perimeter and area of a regular  $n$ -gon and formulas to find interior and exterior angles of a regular  $n$ -gon and their sums.

### G1.6 Circles and Their Properties

- G1.6.1** Solve multistep problems involving circumference and area of circles.
- G1.6.2** Solve problems and justify arguments about chords and lines tangent to circles.
- G1.6.3** Solve problems and justify arguments about central angles, inscribed angles, and triangles in circles.
- G1.6.4** Know and use properties of arcs and sectors and find lengths of arcs and areas of sectors.

### G1.7 Conic Sections and Their Properties

- G.1.7.1** Find an equation of a circle given its center and radius; given the equation of a circle, find its center and radius.

## STRAND 3: GEOMETRY AND TRIGONOMETRY (CONT.)

**G1.7.2** Identify and distinguish among geometric representations of parabolas, circles, ellipses, and hyperbolas; describe their symmetries, and explain how they are related to cones.

**G1.7.3** Graph ellipses and hyperbolas with axes parallel to the  $x$ - and  $y$ -axes, given equations.

### G1.8 Three- Dimensional Figures

**G1.8.1** Solve multistep problems involving surface area and volume of pyramids, prisms, cones, cylinders, hemispheres, and spheres.

**G1.8.2** Identify symmetries of pyramids, prisms, cones, cylinders, hemispheres, and spheres.

## STANDARD G2: RELATIONSHIPS BETWEEN FIGURES

Students use and justify relationships between lines, angles, area and volume formulas, and 2- and 3-dimensional representations. They solve problems and provide proofs about congruence and similarity.

### G2.1 Relationships Between Area and Volume Formulas

**G2.1.1** Know and demonstrate the relationships between the area formula of a triangle, the area formula of a parallelogram, and the area formula of a trapezoid.

**G2.1.2** Know and demonstrate the relationships between the area formulas of various quadrilaterals.

**G2.1.3** Know and use the relationship between the volumes of pyramids and prisms (of equal base and height) and cones and cylinders (of equal base and height).

### G2.2 Relationships Between Two-dimensional and Three-dimensional Representations

**G2.2.1** Identify or sketch a possible three-dimensional figure, given two-dimensional views. Create a two-dimensional representation of a three-dimensional figure.

**G2.2.2** Identify or sketch cross sections of three-dimensional figures. Identify or sketch solids formed by revolving two-dimensional figures around lines.

### G2.3 Congruence and Similarity

**G2.3.1** Prove that triangles are congruent using the SSS, SAS, ASA, and AAS criteria, and that right triangles, are congruent using the hypotenuse-leg criterion.

**G2.3.2** Use theorems about congruent triangles to prove additional theorems and solve problems, with and without use of coordinates.

**G2.3.3** Prove that triangles are similar by using SSS, SAS, and AA conditions for similarity.

**G2.3.4** Use theorems about similar triangles to solve problems with and without use of coordinates.

**G2.3.5** Know and apply the theorem stating that the effect of a scale factor of  $k$  relating one two-dimensional figure to another or one three-dimensional figure to another, on the length, area, and volume of the figures is to multiply each by  $k$ ,  $k^2$ , and  $k^3$ , respectively.

## STRAND 3: GEOMETRY AND TRIGONOMETRY (CONT.)

### STANDARD G3: TRANSFORMATIONS OF FIGURES IN THE PLANE

Students will solve problems about distance-preserving transformations and shape-preserving transformations. The transformations will be described synthetically and, in simple cases, by analytic expressions in coordinates.

#### G3.1 Distance-preserving Transformations: Isometries

- G3.1.1** Define reflection, rotation, translation, and glide reflection and find the image of a figure under a given isometry.
- G3.1.2** Given two figures that are images of each other under an isometry, find the isometry and describe it completely.
- G3.1.3** Find the image of a figure under the composition of two or more isometries and determine whether the resulting figure is a reflection, rotation, translation, or glide reflection image of the original figure.

#### G3.2 Shape-preserving Transformations: Dilations and Isometries

- G3.2.1** Know the definition of dilation and find the image of a figure under a given dilation.
- G3.2.2** Given two figures that are images of each other under some dilation, identify the center and magnitude of the dilation.

#### Recommended:

- \*G1.4.5 Understand the definition of a cyclic quadrilateral and know and use the basic properties of cyclic quadrilaterals.
- \*G1.7.4 Know and use the relationship between the vertices and foci in an ellipse, the vertices and foci in a hyperbola, and the directrix and focus in a parabola, interpret these relationships in applied contexts.
- \*G3.2.3 Find the image of a figure under the composition of a dilation and an isometry.

## STRAND 4: STATISTICS AND PROBABILITY

(S)

In Kindergarten through Grade 8, students develop the ability to read, analyze, and construct a repertoire of statistical graphs. Students also examine the fundamentals of experimental and theoretical probability in informal ways. The Basic Counting Principle and tree diagrams serve as tools to solve simple counting problems in these grades.

During high school, students build on that foundation. They develop the data interpretation and decision-making skills that will serve them in their further study of mathematics as well as in their coursework in the physical, biological, and social sciences. Students learn important skills related to the collection, display, and interpretation of both univariate and bivariate data. They understand basic sampling methods and apply principles of effective data analysis and data presentation. These skills are also highly valuable outside of school, both in the workplace and in day-to-day life.

In probability, students utilize probability models to calculate probabilities and make decisions. The normal distribution and its properties are studied. Students then use their understanding of probability to make decisions, solve problems, and determine whether or not statements about probabilities of events are reasonable. Students use technology when appropriate, including spreadsheets. This strong background in statistics and probability will enable students to be savvy decision-makers and smart information-consumers and producers who have a full range of tools in order to make wise choices.

### STANDARD S1: UNIVARIATE DATA – EXAMINING DISTRIBUTIONS

Students plot and analyze univariate data by considering the shape of distributions and analyzing outliers; they find and interpret commonly-used measures of center and variation; and they explain and use properties of the normal distribution.

#### S1.1 Producing and Interpreting Plots

- S1.1.1** Construct and interpret dot plots, histograms, relative frequency histograms, bar graphs, basic control charts, and box plots with appropriate labels and scales; determine which kinds of plots are appropriate for different types of data; compare data sets and interpret differences based on graphs and summary statistics.
- S1.1.2** Given a distribution of a variable in a data set, describe its shape, including symmetry or skewness, and state how the shape is related to measures of center (mean and median) and measures of variation (range and standard deviation) with particular attention to the effects of outliers on these measures.

#### S1.2 Measures of Center and Variation

- S1.2.1** Calculate and interpret measures of center including: mean, median, and mode; explain uses, advantages and disadvantages of each measure given a particular set of data and its context.
- S1.2.2** Estimate the position of the mean, median, and mode in both symmetrical and skewed distributions, and from a frequency distribution or histogram.
- S1.2.3** Compute and interpret measures of variation, including percentiles, quartiles, interquartile range, variance, and standard deviation.

#### S1.3 The Normal Distribution

- S1.3.1** Explain the concept of distribution and the relationship between summary statistics for a data set and parameters of a distribution.
- S1.3.2** Describe characteristics of the normal distribution, including its shape and the relationships among its mean, median, and mode.
- S1.3.3** Know and use the fact that about 68%, 95%, and 99.7% of the data lie within one, two, and three standard deviations of the mean, respectively in a normal distribution.
- S1.3.4** Calculate z-scores, use z-scores to recognize outliers, and use z-scores to make informed decisions.

## STRAND 4: STATISTICS AND PROBABILITY (CONT.)

### STANDARD S2: BIVARIATE DATA – EXAMINING RELATIONSHIPS

Students plot and interpret bivariate data by constructing scatterplots, recognizing linear and nonlinear patterns, and interpreting correlation coefficients; they fit and interpret regression models, using technology as appropriate.

#### S2.1 Scatterplots and Correlation

- S2.1.1** Construct a scatterplot for a bivariate data set with appropriate labels and scales.
- S2.1.2** Given a scatterplot, identify patterns, clusters, and outliers. Recognize no correlation, weak correlation, and strong correlation.
- S2.1.3** Estimate and interpret Pearson’s correlation coefficient for a scatterplot of a bivariate data set. Recognize that correlation measures the strength of linear association.
- S2.1.4** Differentiate between correlation and causation. Know that a strong correlation does not imply a cause-and-effect relationship. Recognize the role of lurking variables in correlation.

#### S2.2 Linear Regression

- S2.2.1** For bivariate data that appear to form a linear pattern, find the least squares regression line by estimating visually and by calculating the equation of the regression line. Interpret the slope of the equation for a regression line.
- S2.2.2** Use the equation of the least squares regression line to make appropriate predictions.

### STANDARD S3: SAMPLES, SURVEYS, AND EXPERIMENTS

Students understand and apply sampling and various sampling methods, examine surveys and experiments, identify bias in methods of conducting surveys, and learn strategies to minimize bias. They understand basic principles of good experimental design.

#### S3.1 Data Collection and Analysis

- S3.1.1** Know the meanings of a sample from a population and a census of a population, and distinguish between sample statistics and population parameters.
- S3.1.2** Identify possible sources of bias in data collection and sampling methods and simple experiments; describe how such bias can be reduced and controlled by random sampling; explain the impact of such bias on conclusions made from analysis of the data; and know the effect of replication on the precision of estimates.
- S3.1.3** Distinguish between an observational study and an experimental study, and identify, in context, the conclusions that can be drawn from each.

### STANDARD S4: PROBABILITY MODELS AND PROBABILITY CALCULATION

Students understand probability and find probabilities in various situations, including those involving compound events, using diagrams, tables, geometric models and counting strategies; they apply the concepts of probability to make decisions.

#### S4.1 Probability

- S4.1.1** Understand and construct sample spaces in simple situations.
- S4.1.2** Define mutually exclusive events, independent events, dependent events, compound events, complementary events and conditional probabilities; and use the definitions to compute probabilities.



## STRAND 4: STATISTICS AND PROBABILITY (CONT.)

### S4.2 Application and Representation

- S4.2.1** Compute probabilities of events using tree diagrams, formulas for combinations and permutations, Venn diagrams, or other counting techniques.
- S4.2.2** Apply probability concepts to practical situations, in such settings as finance, health, ecology, or epidemiology, to make informed decisions.

#### RECOMMENDED:

- \*S3.1.4** Design simple experiments or investigations to collect data to answer questions of interest; interpret and present results.
- \*S3.1.5** Understand methods of sampling, including random sampling, stratified sampling, and convenience samples, and be able to determine, in context, the advantages and disadvantages of each.
- \*S3.1.6** Explain the importance of randomization, double-blind protocols, replication, and the placebo effect in designing experiments and interpreting the results of studies.
- \*S3.2.1** Explain the basic ideas of statistical process control, including recording data from a process over time.
- \*S3.2.2** Read and interpret basic control charts; detect patterns and departures from patterns.
- \*S4.1.3** Design and carry out an appropriate simulation using random digits to estimate answers to questions about probability; estimate probabilities using results of a simulation; compare results of simulations to theoretical probabilities.



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